sults of the machine and analytic calculations are shown in Table 1

These results show that the important body parameter is the loading W/C_LA Comparison of the two cases with re-entry velocity 1 2 times circular velocity and W/C_DA = 58 82 and 121 95 psf, shows that, since W/C_LA is the same for both, the critical angles are the same even though the values of C_L differ appreciably (−0 2 and −1 0)

References

[1] Kornreich, T , ' Approximate analytic solutions for the range of a nonlifting re-entry trajectory,'' AIAA J 1, 1925–1926 (1963)

[2] Wang, K and Ting, L , ''Analytic solutions of planar re-entry trajectories with lift and drag, Polytech Inst Brooklyn Aerodynamics Lab PIBAL Rept 601, Air Force Office Sci Res AFOSR TN 60-508 (April 1960)

Technical Comments

Comment on "An Approximate Solution for Laminar Boundary Layer Flow"

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In a recent paper[1] Kosson presented an approximate solu tion for two-dimensional, incompressible, laminar bound ary-layer flow with an arbitrary pressure gradient As an example of his method Kosson considered the external flow $U = 2U_\infty \sin(x/R)$ past a circular cylinder of radius R Kosson compared the results obtained by his method with a series solution given by Ulrich[2] using, presumably, the more accurate values for the coefficients of the terms of the series obtained by Tifford [3] However, an exact numerical solution for the flow has been obtained by Terrill,[4] and the results given by Kosson for nondimensional skin friction, displacement thickness, and momentum thickness are compared with Terrill's results in Table 1

Kosson points out that "a higher-order polynomial is required in order for the series expansion method to be valid " and that part of the discrepancy in the region of decelerating flow "may be attributed to errors in the series solution " The reason for the slow convergence of the series expansion near separation is that, for this external flow, there is a singularity in the laminar boundary-layer equations at the separation point (discussed in Ref 4) Nevertheless, it can be seen from Table 1 that there is not a great difference between the series and the exact solutions at an angle of 100° from the leading edge Near separation the skin friction behaves like $\xi^{1/2}$, where ξ is the distance from separation, and so decreases very rapidly as the separation point is approached It is not surprising that a series method, for which the skin friction is almost certain to fall less rapidly than for an exact numerical solution (because of the singularity), predicts separation later than 104 45° It is more surprising that Kosson's solution gives separation before the correct value; this indicates that Kosson's values for the skin friction are much less than the true values near separa tion and is confirmed by the results at η = 100° However, there appears to be good agreement between his re sults and the exact results for the displacement and momentum thicknesses at the separation point

References

[1] Kossom, L R , 'An approximate solution for laminar boundary layer flow,' AIAA J 1, 1088–97 (1963)

[2] Ulrich, A , ' Die Laminare Reibungsschicht am Kreiszylinder,'' Z Deut Luftfahrtforsch F B 1762 (1943)

[3] Tifford, A N , ''Heat transfer and frictional effects in

Table 1 Results for the external flow $U = 2U_\infty \sin(x/R)$

	$\frac{\tau_0}{\rho U_\infty^2}\left(\frac{U_\infty R}{\nu}\right)^{1/2}$			$\delta^*\left(\frac{U_\infty}{\nu R}\right)^{1/2}$			$\theta\left(\frac{U_\infty}{\nu R}\right)^{1/2}$		
η°	Kosson	Series	Exact	Kosson	Series	Exact	Kosson	Series	Exact
0	0	0	0	0 456	0 46	0 458	0 203	0 21	0 207
30	1 62	1 64	1 64	0 481	0 49	0 485	0 212	0 22	0 218
60	2 22	2 26	2 25	0 580	0 59	0 585	0 250	0 26	0 26
90	1 26	1 35	1 35	0 918	0 89	0 89	0 357	0 35	0 36
100	0 34	0 71	0 64	1 372	1 12	1 22	0 443	0 40	0 44
102 45	0			1 758			0 472		
104 45			0			1 704			0 484
108 8		0			1 45			0 40	

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laminar boundary-layers Part 4: Universal series solutions," Wright Air Dev Center TR 53-288 (1954)

[4] Terrill, R M , "Laminar boundary-layer flow near separation with and without suction," Phil Trans Roy Soc (London) **A253,** 55–100 (1960)

Reply by Author to R M Terrill

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THE writer had not been aware of Terrill's exact solution[1] for sinusoidal velocity distribution and is pleased at the closeness of agreement of the approximate solution[2] with it

In dealing with the approximate solution near the separation point, it should be noted that a transition from use of the polynomial inner solution [Eqs (33–37)] to use of the von Kármán Millikan inner solution [Eqs (29–32)] takes place when the velocity profile inflection point occurs at a dimensionless stream-function value z greater than 0 15 A small discontinuity in the boundary-layer parameters is

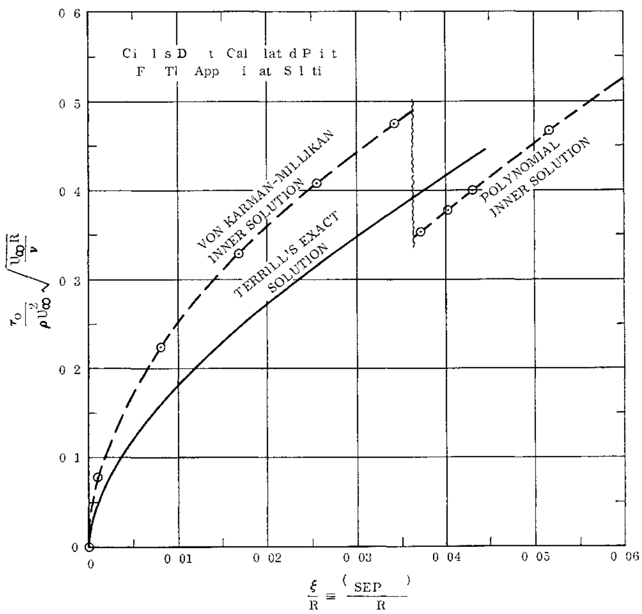

Fig 1 Shear stress near separation point, sinusoidal velocity distribution, $U_a = 2U_\infty \sin(X/R)$

associated with this transition, as shown in Table 1 for the sinusoidal velocity distribution Note that the nondimensional shear-stress value reported in Table 4 of Ref 2 for $\eta = 100°$ was in error, and should be 0 40

Although the approximate solution errs in predicting separation too soon (in this example), the variation of shear stress with distance from the separation point is similar to that calculated by Terrill, as shown in Fig 1 The good agreement in displacement and momentum thickness at the separation point is related to this

Also, it may be noted that the value of $C = 0\ 788$ used in the approximate solution is, to some extent, arbitrary and was determined at the forward stagnation point It is possible that a different value of C would give better agreement in the vicinity of the separation point

Table 1 Approximate solution results for sinusoidal velocity distribution

η	Inner solution	$\frac{\tau_0}{\rho U_\infty^2}\left(\frac{U_\infty R}{\nu}\right)^{1/2}$	$\delta^*\left(\frac{U_\infty R}{\nu}\right)^{1/2}$	$\theta\left(\frac{U_\infty R}{\nu}\right)^{1/2}$
0	Polynomial	0	0 456	0 203
30	"	1 62	0 481	0 212
60		2 22	0 580	0 250
90		1 26	0 918	0 357
100		0 40	1 372	0 443
100 33		0 35	1 409	0 447
100 5	von Kármán-Millikan	0 47	1 328	0 443
101	"	0 41	1 376	0 449
101 5		0 33	1 438	0 456
102		0 22	1 527	0 463
102 40		0 08	1 671	0 471
102 46		0 00	1.759	0.472

References

[1] Terrill, R M , "Laminar boundary-layer flow near separation with and without suction " Phil Trans Roy Soc (London) **A253,** 55–100 (1960)

[2] Kosson, R L , "An approximate solution for laminar boundary layer flow,' AIAA J **1,** 1088–1096 (1963)

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Comment on "A Class of Linear Magnetohydrodynamic Flows"

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IN a recent note[1] the characteristics of steady-state magnetohydrodynamic flows of an incompressible, electrically conducting fluid with constant scalar properties were examined with respect to the functional dependence of the solutions that would be necessary to linearize the governing equations Two cases were specifically considered and solutions presented Closer examination of the problem shows that the results obtained are more restrictive than is immediately obvious, and the following analysis is presented to show exactly the limitations implied by the interactions among the assumptions in each of the two cases The symbols have their usual meaning, and the mks system of units has been used

In case 1, the form of the velocity and magnetic field [$\mathbf{V} = \mathbf{i}u(y,z)$ and $\mathbf{B} = \mathbf{i}B_x(y,z) + \mathbf{k}B_0$] is such that the continuity equation and the divergence relation for $\mathbf{B}$ are identically satisfied An expansion of Ampere's law shows that $J_x = 0$, and a subsequent expansion of Ohm's law shows that $E_x = 0$ Eliminating the current between Ohm's law and the divergence relation, and acknowledging the divergence relation for $\mathbf{E}$ gives the result $\partial u/\partial y = 0$ or, integrating, $u = u(z)$ This shows that the form of the assumed relation for the velocity is too general to be compatible with the functional forms of the other variables

The momentum equation may now be expanded to give

$$\partial P/\partial x = B_0 J_y + \mu(d^2u/dz^2) \qquad (1)$$

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